Eur. Phys. J. B **4**, 121–129 (1998)

THE EUROPEAN PHYSICAL JOURNAL B

Velocity fluctuations of a column of water streaming down a wire: the possibility of one-dimensional turbulence

H. Kellay[a] and J. Rouch

Centre de Physique Moléculaire Optique et Hertzienne[b], Université Bordeaux I, 351 cours de la Liberation, 33405 Talence Cedex, France

Received: 1 October 1997 / Revised: 29 January 1998 / Accepted: 3 March 1998

Abstract. The results of measurements of the velocity fluctuations of a thin column of water streaming down a wire show that the dynamics of this process is turbulent at high injection flux and is characterized by a Kolmogorov like spectrum for the energy density. This density $E(k)$, proportional to the power spectrum of the longitudinal velocity, scales as $k^{-\beta}$ with $\beta = 1.66 \pm 0.05$ reminiscent of Kolmogorov turbulence in three-dimensions. It is suggested that this system can be modeled as a quasi one-dimensional compressible system where the compressibility stems from the instability of the column to thickness fluctuations.

PACS. 47.20.Ma Interfacial instability – 47.27.-i Turbulent flows, convection, and heat transfer – 47.40.-x Compressible flows; shock and detonation phenomena

Introduction

Interest in the study of turbulence in dimensions lower than three stems from the fact that the interplay between coherent structures in the flow and the small scale fluctuations can be better understood. The three-dimensional case seems to be the most complex. The two-dimensional case is supposedly easier and somewhat better understood, but the one-dimensional case is believed to be the most promising since the coherent structures, shocks in this case, are well understood and their interaction with the fluctuating background is tractable [1–4].

In an attempt to produce low dimensional turbulence in the laboratory, we chose to study the streaming of a thin column of water down a wire. Herein, we report the results of measurements of the velocity fluctuations of this column. Despite the large amount of work concerning the stability of a liquid column, to our knowledge, there have been no experiments in which the velocity fluctuations were measured. The water column, confined to run down a wire in our case, is smooth near the entrance but exhibits large fluctuations of its thickness downstream and visual observations show that the fluid patches stream down at different velocities making the flow look turbulent. The results reported here show that at high injection flux, in the rough film regime seen downstream from the entrance, the amplitude of the power spectrum of the longitudinal velocity fluctuations is at least an order of magnitude larger than the amplitude of the transverse velocity spectrum. The longitudinal spectrum obeys a scaling law as a function of the frequency. By invoking the Taylor frozen turbulence assumption, the measurements indicate that the one-dimensional energy density spectrum, $E(k)$, (proportional to the power spectrum of the longitudinal velocity (along the wire)), where k is the wave number in the longitudinal direction, scales as $k^{-\beta}$ with β values of about 1.66 ± 0.05. This is very close to the Kolmogorov exponent ($\beta = 5/3$) predicted long ago for three-dimensional turbulence. However, it is highly unlikely that this system can sustain fully developed three-dimensional (3d) turbulence since in our case the fluid is confined to run down a wire and the diameter of the column remains small (less than 0.1 cm). Velocity fluctuations perpendicular to the wire are heavily damped because of friction against the wire, also the fluctuations of the transverse velocity of the column are small compared to the longitudinal velocity fluctuations. The result is closer to what has been found recently for the one-dimensional Burgers equation driven with a particular noise [1,2]. This analogy is further motivated by the results of measurements of the probability density function (pdf) of velocity increments as well as the variation of the moments of these increments as a function of spatial separation. These moments starting with the third moment vary linearly as a function of the separation while the second moment shows a weaker dependence in agreement with recent simulations and theories [1,2]. Also, the shape of the pdf revealed a strong asymetry between the positive increments and the negative ones. In particular, the positive part of the pdf can be well approximated by an exponential function while the negative part can be well approximated by a power law dependence in close analogy with recent simulations and theories [1,3]. It is suggested that the system studied here can

[a] e-mail: kellay@iris.cpmoh.u-bordeaux.fr
[b] CPMOH is URA 283 of the CNRS.

be modeled as a compressible one-dimensional (1d) fluid in a first approximation. The relevent equations governing this effective 1d problem are given.

Experimental

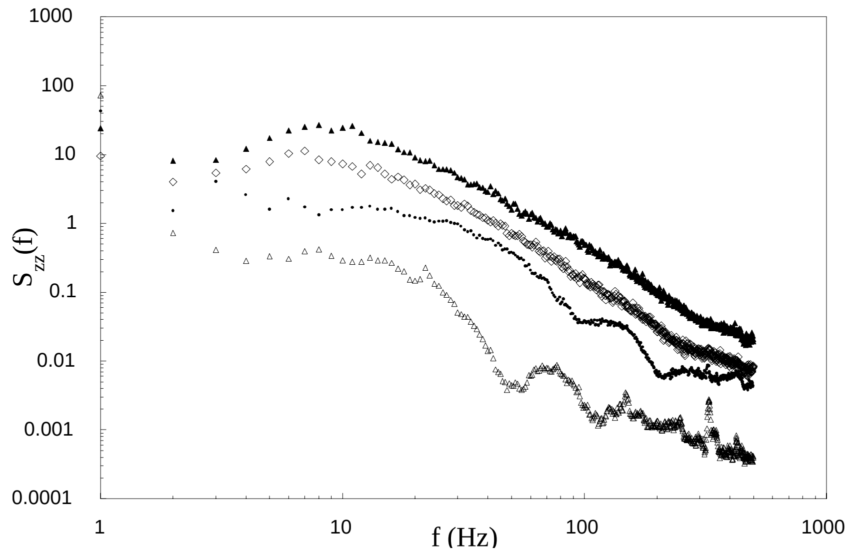
Our experimental setup is simple and consists of a micropump with variable flow rate that pumps water from a reservoir to the top of the wire. A flexible tube of small diameter (0.2 cm) brought this water to the top of the wire. The water emanating from the tube was brought to touch the top of the wire suspended to a small hook. This water forms a thin film around the wire and streams down at speeds controlled by the injection flow rate. The total diameter of the column near the entrance is about 0.1 cm. In order to have a smooth film of water around the wire near the entrance and therby minimizing noise due to the injection of the fluid, a little detergent was added to the water to improve the wetting of the wire. Careful adjustements of the angle between the tube and the wire were also necessary to have a smooth entrance. When no surfactant is added to the water we found it difficult to make the film at the entrance very smooth. We added a few weight percent of detergent to the water for the experiments reported here and checked that small variations of the quantity of detergent did not affect the results measurably in the rough film regime as long as the film at the entrance is kept relatively smooth. For the results reported here, the wire was made of nylon (fishing wire with a diameter of 0.04 cm and a length of 150 cm). The flow rate, controlled by the variable speed micropump, could be varied from less than 0.1 ml/s to 1 ml/s. For higher flow rates the column was difficult to stabilize as drops start to detach from it. At the highest flow rate used (1 ml/s), the average velocity of the flowing film is estimated to be 200 cm/s for a total column diameter of 0.1 cm.

The film of water streaming down the wire is known to be unstable with respect to undulations of the film thickness. Further down the wire the column of water breaks up into droplets. At flow rates less than 0.1 ml/s the flow is in the form of drops that stream down in an almost periodic fashion as in a dripping faucet. As the flux is increased above 0.1 ml/s, a smooth thin film forms at the entrance, but is unstable to drop formation further down the wire. Between the smooth film regime near the entrance and the drop regime far away from the entrance, a rough film regime is seen where the column exhibits large thickness fluctuations. This rough film region can be seen from distances of about 20 cm from the entrance down to distances of 70 cm and more for the high flow rates. The point where the rough film breaks up into drops depends on the flow rate; the higher the flux the further is the point of break up.

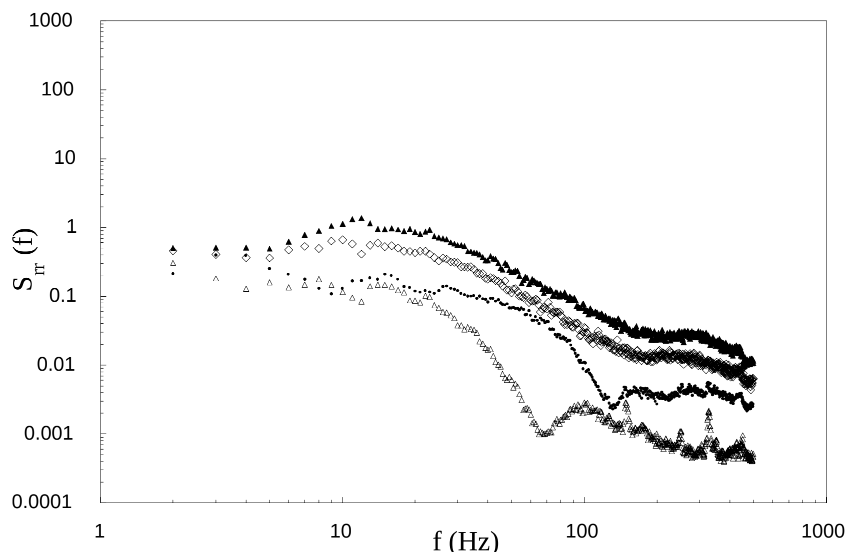
An optical fiber velocimeter was used to measure the velocity fluctuations. Such a velocimeter has been developed recently and used to study velocity fluctuations in turbulent soap films [5]. The method permits one to detect velocity fluctuations both in the longitudinal and transverse directions almost simultaneously making it very suitable for the measurement of these fluctuations in systems with reduced spatial dimension. The principle of the method is simple and relies on the measurement of the deflection of a short and very thin optical fiber the tip of which penetrates the flow. The deflection of the fiber is proportional to the fluid velocity. In the experiment considered here, the optical fiber is set right against the streaming column of water, without touching the wire but in contact with the flowing water. The optical fiber, coupled to a HeNe laser, has a tip from which a light beam is emited. The fluctuations in position of this tip are measured using a position detector capable of detecting the deflections of the fiber both in the longitudinal and transverse direction. The output of the detector is read by a computer equiped with an AD converter. The time trace of the position fluctuations of the fiber tip was measured and its power spectrum was calculated. It is necessary to recall that the optical fiber, which plays the role of a mechanical vibration detector, can be used only up to frequencies below its natural resonance frequency [5], which was about 800 Hz in this experiment. This frequency depends on the diameter of the fiber and its length. These were fixed in such a way as to have a good signal to noise ratio and a relatively high resonance: we used a length of about 0.8 cm (an order of magnitude larger than the radius of the column to minimize the effect of the variation of the column radius) and a diameter of 60 micrometers (an order of magnitude less than the radius of the column to minimize the perturbation of the flow by the fiber).

The power spectrum of the time trace of the position fluctuations of the fiber tip or equivalently the velocity fluctuations is shown in Figures 1 and 2 for low and high flow rates respectively. These measurements are taken at a fixed distance of 50 cm from the entrance and for different flow rates. At this location and for flow rates less than 0.2 ml/s one can still observe drop like disturbances. At higher flow rates, the drops start to coalesce to form larger patches until a rough film forms for flow rates of 0.25 ml/s and higher.

For small flow rates (Fig. 1a), the longitudinal velocity spectrum, $S_{zz}(f) \equiv \langle v_z(f) v_z^*(f) \rangle$ (where z is in the longitudinal direction, $v_z(f)$ is the Fourier transform of the longitudinal velocity and $v_z^*(f)$ is its complex conjugate, f is the frequency, and the brackets denote time or ensemble averaging), decays rapidly above frequencies of about 100 Hz and seems to have a few characteristic frequencies as evidenced by some broad peaks. As the flow rate is increased above a flow rate of 0.1 ml/s, the spectrum becomes broad-band and the peaks disappear as can be seen for a flow rate of 0.2 ml/s in Figure 1a. For the low flow rates (less than 0.2 ml/s) and for intermediate flow rates (less than 0.4 ml/s) there is no obvious scaling. As the flow rate is increased a small region of the spectrum starts to show scaling (flow rate 0.4 ml/s); this frequency region widens as the flow rate increases and above flow rate 0.4 ml/s scaling is observed for most of the frequency range covered. At high flow rates (Fig. 2a), the longitudinal velocity spectra show a scaling law as a function of the frequency. This scaling law, valid for a



(a)



(b)

Fig. 1. a) Longitudinal velocity spectra $S_{zz}(f) \equiv \langle v_z^2(f) v_z^*(f) \rangle$ (where z is in the longitudinal direction, $v_z(f)$ is the Fourier transform of the longitudinal velocity, $v_z^*(f)$ is its complex conjugate, and f is the frequency) for low flow rates: 0.02 (open triangles), 0.05 (dots), 0.20 (open diamonds), and 0.3 (filled triangles) ml/s. The spectra are multiplied by 10^5. b) Transverse velocity spectra $S_{rr}(f) \equiv \langle v_r^2(f) v_r^*(f) \rangle$ (where r is in the radial or transverse direction, $v_r(f)$ is the Fourier transform of the transverse velocity, and $v_r^*(f)$ is its complex conjugate) for the same flow rates: 0.02 (open triangles), 0.05 (dots), 0.20 (open diamonds), and 0.3 (filled triangles) ml/s. The spectra are multiplied by 10^5.

little more than a decade, persists for all flow rates above a certain value (flow rate 0.4 ml/s). Power law fits to the data gave exponents between -1.6 and -1.7. In Figure 3, examples of such power law fits to the spectra are shown (flow rates 0.8 and 0.9 ml/s); the exponents measured are -1.66 and -1.63 respectively. This scaling law can be seen for frequencies between about 30 Hz and 500 Hz. At low frequencies the amplitude decreases and there seems to be a broad peak in the spectrum at frequencies of about 20 Hz. Above 500 Hz, the effect of the resonance of the fiber comes into play. This resonance as mentioned above was fixed at about 800 Hz.

The transverse velocity spectrum, $S_{rr}(f) \equiv \langle v_r(f) v_r^*(f) \rangle$ (where r is in the radial or transverse direction, $v_r(f)$ is the Fourier transform of the transverse velocity, and $v_r^*(f)$ is its complex conjugate), also shows a few peaks at low flow rates as seen in Figure 1b. This spectrum becomes broad band at frequencies below 100 Hz for higher injection rates (Fig. 2b). A broad peak appears at about 200 Hz. The amplitude of the

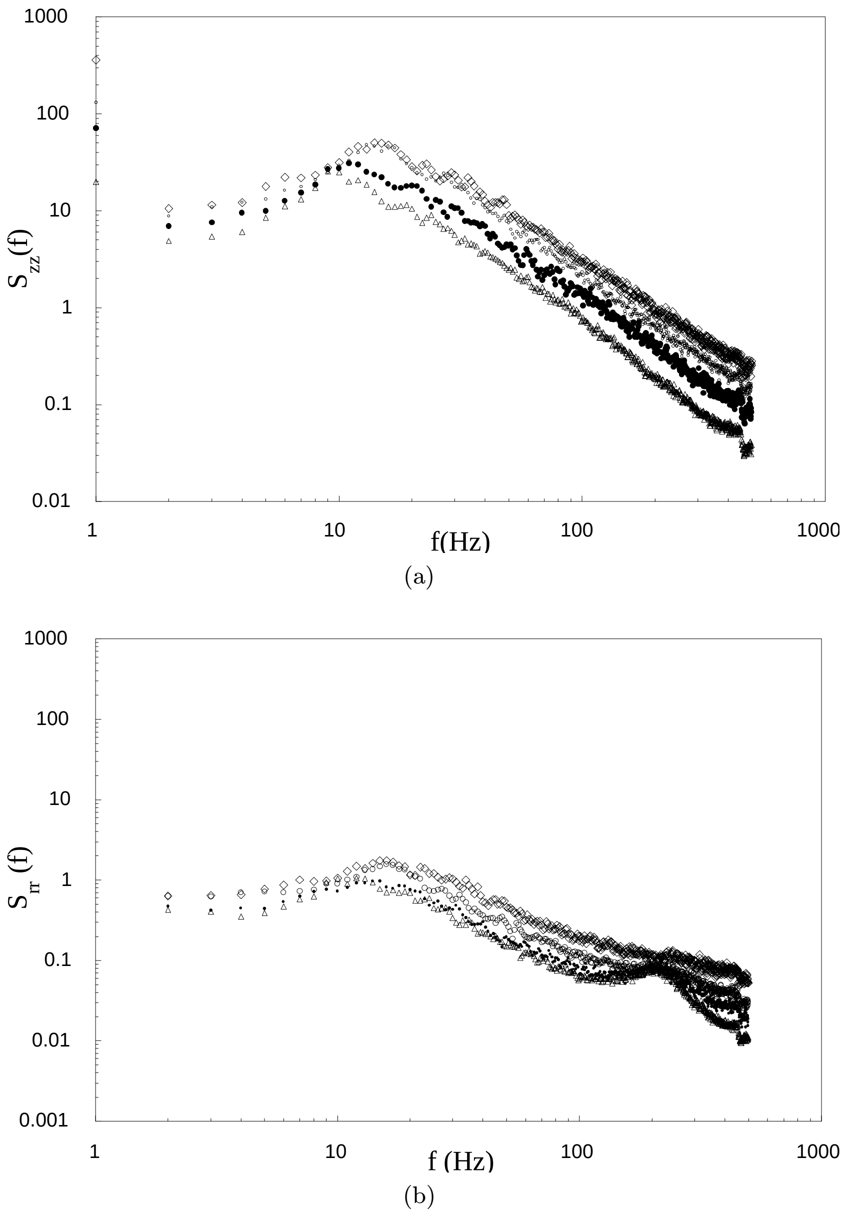


Fig. 2. a) Longitudinal velocity spectra for high flow rates: 0.40 (open triangles), 0.50 (dots), 0.80 (open cirles), and 0.90 (open diamonds) ml/s. The spectra are multiplied by 10^5. b) Transverse velocity spectra for the same flow rates: 0.40 (open triangles), 0.50 (dots), 0.80 (open cirles), and 0.90 (open diamonds) ml/s. The spectra are multiplied by 10^5.

transverse velocity spectrum is seen to be at least an order of magnitude below the amplitude of the longitudinal spectrum. At high flow rates the transverse fluctuations are then much smaller than the longitudinal ones. These transverse fluctuations are most probably due to the thickness fluctuations of the film and may be a measure of the rate of change of the film thickness. The spectra at high flow rates show a small range where scaling can be discerned with an exponent between -1 and -1.4 as seen in Figure 3 where examples of power law fits are shown (flow rates of 0.8 and 0.9 ml/s). However, since the range is small this scaling may not be significant.

Since the optical fiber may perturb the flow and since its deflection may be sensitive to the thickness fluctuations of the column, we have carried out additional measurements using a commercial Laser Doppler Velocimeter (DISA 55). Laser Doppler Velocimetry (LDV) is a standard technique for the measurement of velocity and descriptions of its principles can be found in several references. LDV is a non intrusive optical method which measures the velocity of seed particles in the fluid as they stream past a system of optical fringes obtained at the intersection of two coherent laser beams with the same polarization. The measuring volume was set very close to the wire with the fringes planes perpendicular to the flow

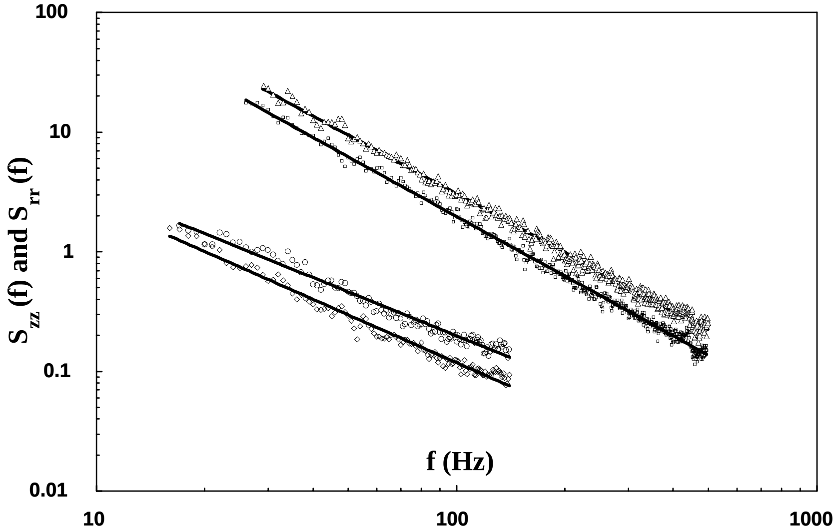


Fig. 3. Examples of power law fits to longitudinal and transverse velocity spectra: flow rates 0.8 (open squares and open diamonds) and 0.9 (open triangles and open circles) ml/s. The exponents are −1.66 (0.8 ml/s) and −1.63 (0.9 ml/s) for the longitudinal spectra and −1.32 (0.8 ml/s) and −1.22 (0.9 ml/s) for the transverse spectra.

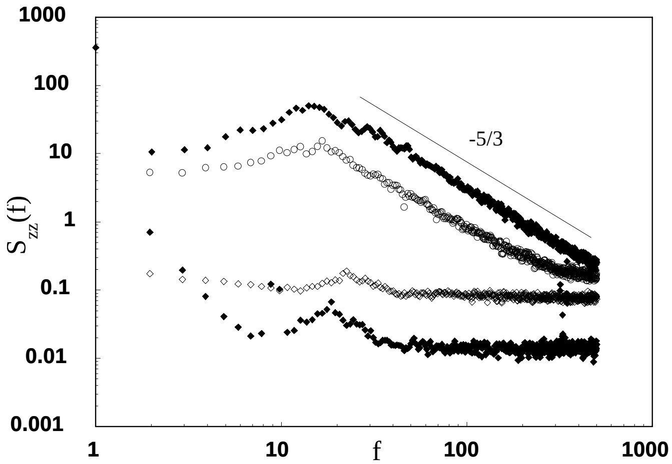


Fig. 4. Comparison between LDV spectra and Fiber Velocimeter spectra in the smooth film regime and the rough film regime. The two upper spectra are taken at a distance of 50 cm from the entrance at a flow rate of 0.9 ml/s (LDV spectrum: open circles, fiber velocimeter spectrum filled diamonds). The two lower spectra are for small flow rates at 8 cm from the entrance; filled diamonds: fiber spectrum for a flow rate of 0.12 ml/s, open diamonds: LDV spectrum for a flow rate of 0.15 ml/s. The spectra have been shifted vertically from each other for better presentation.

direction to measure the longitudinal velocity. This velocity signal was sampled at a frequency of 1000 Hz which was systematically smaller than the count rate provided by the LDV which was controlled in part by the seeding density of the fluid. This count rate was systematically larger than 2000 counts per second and ranged between 2000 and 5000. The time trace thus obtained is Fourier transformed to obtain the power spectrum of the longitudinal velocity fluctuations. Before doing the measurements we checked that the effect of refraction of the beams by the free surface of the column did not affect the measurements. Since our LDV is equipped with a Bragg cell for frequency shifting, the fringes move at a velocity fixed by the frequency shift and the fringe spacing. When the LDV system is used on an immobile object, one measures this velocity. Measurements of this velocity from the wire without the column and with the column of water (with no seed particles) streaming down show that this velocity is unchanged by the presence of the column. This indicates that the fringe pattern and the fringe spacing is not distorted by refraction from the free surface assuring us that such effects are negligeable.

Figure 4 shows an example of a comparison between spectra of the longitudinal velocity measured by the LDV technique and spectra of the longitudinal fluctuations of the fiber position. The two upper spectra are taken in the rough film regime at a distance of 50 cm from the entrance for a flow rate of 0.9 ml/s. The two lower spectra are taken near the entrance for a small flow rate in the smooth film regime. Note that the spectra are quite similar except at low frequencies (below about 10 Hz) which may be due to mechanical vibrations of the experimental set up. Also the velocity spectrum for the high flow rate (measured using LDV) flattens at frequencies above 200 Hz, but this may be due to the low quality of the signal at high frequencies. Overall, however, the spectra convey the same information. For the low flow rate near the entrance where the film is smooth, the spectrum is flat for the high frequencies and shows a small but broad peak at around 20 Hz. In the rough film regime for the high flow rate away from the entrance, the same scaling of the velocity power spectrum is observed for frequencies above 20 Hz.

The scaling observed for the high flow rates at a location of 50 cm from the entrance can be seen at other locations down the column. Figure 5 shows the dependence of the power spectrum of the longitudinal velocity on the distance from the entrance. The spectra are taken for a fixed flow rate of 0.65 ml/s at different distances from the entrance. The spectra very close to the entrance show a broad peak for frequencies below 100 Hz and little high frequency dynamics. As the distance is increased the spectrum broadens and starts to show scaling at distances greater than 22 cm. This scaling behavior can be seen

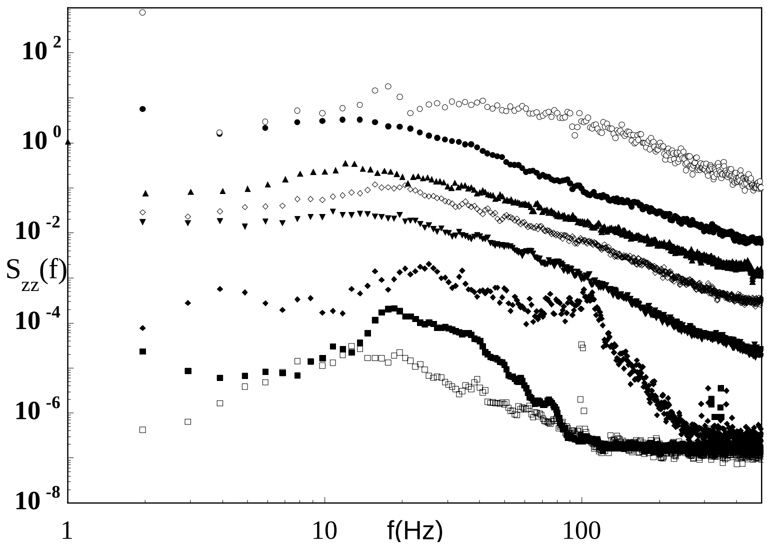


Fig. 5. Spectra taken at a fixed flow rate (0.65 ml/s) and different locations down the wire: a) open squares 5 cm, b) filled squares 8 cm, c) filled diamonds 14 cm, d) inverted triangles 22 cm, e) open diamonds 29 cm, f) filled triangles 50 cm, g) filled circles 75 cm, h) open circles 85 cm. The spectra have been shifted vertically from each other for better presentation.

from distances of about 25 cm to 75 cm; this is basically the rough film region of the flow down the wire. Further down where the rough film starts to break up into drops, the dependence on frequency becomes steeper at higher frequencies and the spectrum flattens at low frequencies starting at about 90 Hz as can be seen for a distance of 85 cm from the entrance.

These measurements at different locations down the column are typical of flow rates above 0.4 ml/s and show that in the rough film regime the longitudinal velocity scaling is quite robust. This scaling does not exist far away from the entrance where the rough film starts to break into drops nor in the smooth film regime close to the entrance where the velocity fluctuations and the column thickness fluctuations are not fully developed. The scaling of the velocity is seen for flow rates above about 0.4 ml/s and at locations between 25 and 75 cm from the entrance.

The results obtained in this experiment did not depend on the diameter of the wire as two other diameters were used: 0.03 cm and 0.08 cm. The diameter of the wire changes the threshold flow rate for scaling as this flow rate increases as the diameter of the wire increases. We have also used metallic wires instead of nylon wires with no change in the obtained results. An important parameter in this experiment was the noise very near the entrance: if the film at the entrance is not smooth the spectra can change considerably as the column shows some asymetry and has a tendency to drop formation. This effect was minimized as much as possible by adding detergent to the water as mentioned above which limited us in the study of the effect of surface tension as we could not vary it considerably (by adding more or less surfactant to the water) without making the entrance noisy. Tests using as little detergent as possible (in the limit where the film at the entrance is relatively smooth) showed similar behavior for the velocity spectra in the rough film regime however. Also, while small variations of the viscosity did not change the scaling results we found that when the viscosity is increased noticeably (by adding glycerol to the water) drop formation occured more readily even close to the entrance.

In addition to these spectral measurements, we carried out a series of measurements of the moments of velocity increments $\delta v(r) = v(z + r) - v(z)$ for the longitudinal component of the velocity in the rough film regime. These increments were calculated from a time series of the velocity as measured using the fiber velocimeter. For this series of measurements the resonance of the fiber was pushed to above 1000 Hz. From the time series one can calculate the velocity differences across a temporal increment δt as $\delta v(\delta t) = v(t) - v(t+\delta t)$; this increment can be transformed into a velocity difference across a scale r where r is related to δt by the frozen turbulence assumption as $r = V\delta t$, where V is the mean flow velocity (about 200 cm/s for the results presented).

Figure 6 shows the probability density function (pdf) of these velocity increments for two separations within the inertial range (where the $-5/3$ scaling is observed): Figure 6a is for a separation of 0.4 cm while Figure 6b is for a separation of 0.6 cm. Note that this pdf which is plotted in a semilogarithmic plot shows a strong asymetry between the negative and positive parts of the increments. As it can be seen from the graphs, the positive velocity increments show a rapid decrease and an exponential tail for the pdf at large velocity increments; while as seen from Figures 7a and b, the negative velocity increments are characterized by a power law tail. A steep decrease is seen for the large velocity increments. Next, we show in Figure 8, the variation of various moments of the absolute value of these increments as a function of the sepration r. Note that all moments starting from the third moment show roughly the same slope on the log-log plot. This slope is consistent with an exponent of 1 for all these high order moments. However, the second moment shows a smaller slope of about 0.5. All the moments show a weaker dependence at distances r greater than 2 cm corresponding to a temporal increment of 10 ms as can be seen in Figure 8.

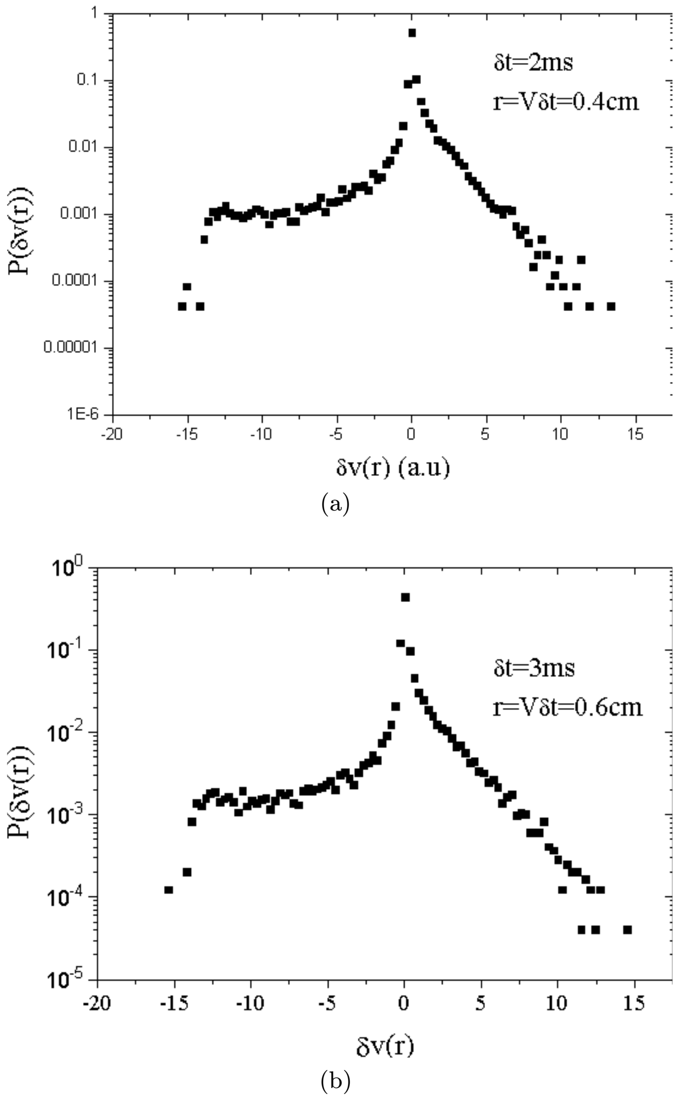


Fig. 6. Probability density function of velocity increments for a spatial separation of 0.4 cm (a) and 0.6 cm (b). The temporal separation is in ms which can be converted to a spatial separation r through the frozen turbulence assumption as: $r = Vdt$ where V (200 cm/s) is the mean velocity of the flow.

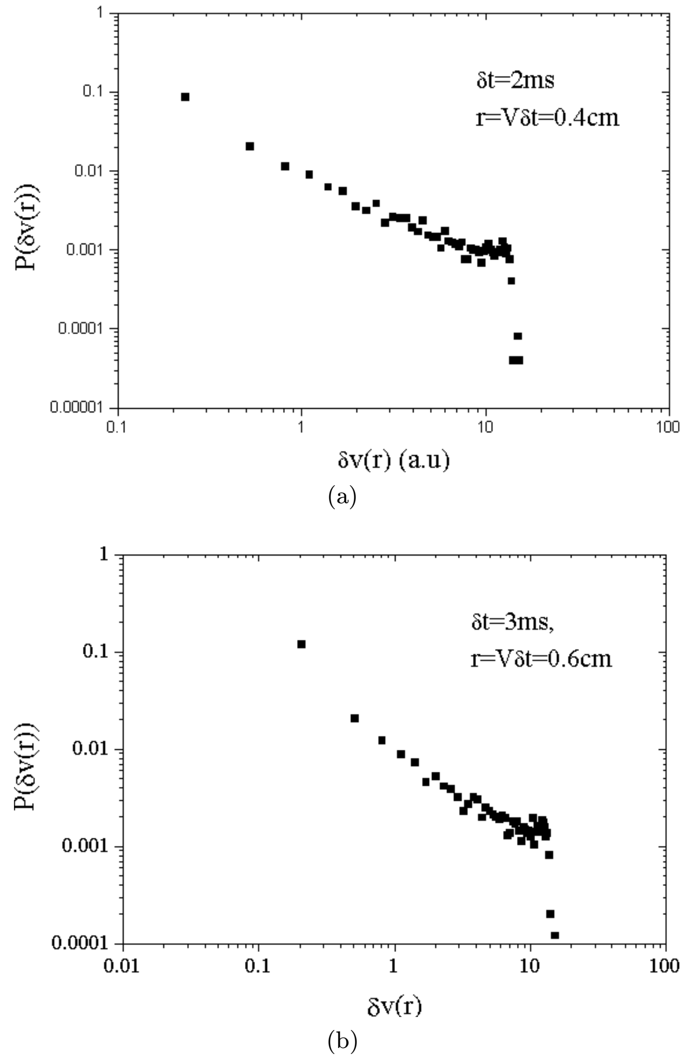


Fig. 7. Negative part of the probability distribution function of velocity increments for the same scales as Figure 6, namely 0.4 cm (a) and 0.6 cm (b).

Discussion

In order to make contact with theory, it is usually necessary in turbulence studies to invoke a hypothesis due to Taylor as invoked above which permits one to transform a frequency (or time scale) to a length scale. This is known as the Taylor frozen turbulence assumption which states that the eddies are being swept across the point of observation without suffering much change in their structure. In practice this assumption is used to calculate a length scale from a time scale in the following way: $k/2\pi = f/V$ where V is the mean flow speed, f is the frequency, and k is a wave number in the direction of the flow. Assuming that this assumption is not violated in such problems of reduced dimensionality, one can transform the temporal information into a spatial information. The length scales covered in this experiment would then range from a few millimeters to several centimeters. More precisely, for the high flow rates in the scaling regime, the mean velocity of the flow being about 200 cm/s, a frequency of 500 Hz corresponds to a scale of 0.4 cm and a frequency of 10 Hz corresponds to a scale of 20 cm. Note that the smallest scale resolved is larger than both column radius and fiber diameter. Using this assumption, the one-dimensional energy density spectrum $E(k) \sim S_{zz}(k = 2\pi f/V)$ shows a scaling law reminiscent of Kolmogorov turbulence. The exponent measured is very close to the Kolmogorov spectrum predicted long ago for three-dimensional turbulence for which $E(k)$ scales as $k^{-5/3}$. In the experiment presented here, it is highly improbable that the turbulence is fully developed and three dimensional. The thickness of the film of water is small and velocity fluctuations in the radial direction (perpendicular to the wire) are highly

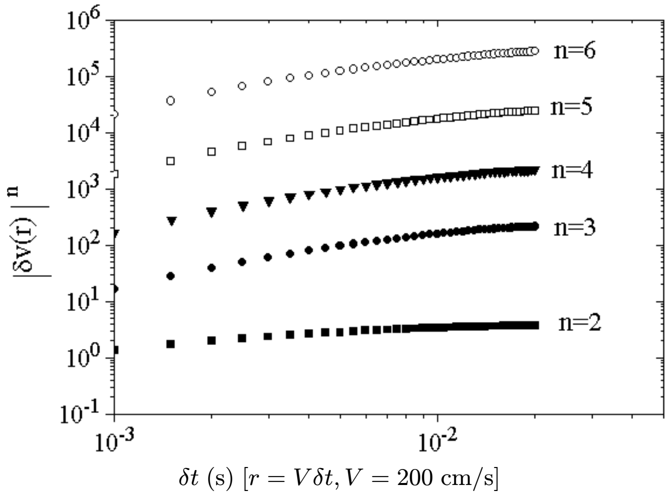


Fig. 8. Moments of the absolute value of velocity increments as a function of the scale r. Here the horizontal scale is the time separation in ms which can be converted to a spatial separation using the frozen turbulence assumption: $r = V\delta t$ ($V = 200$ cm/s). We present these moments from the second up to the sixth moment. Notice that moments starting with the third moment have a slope consistent with a value of 1 while the second moment has a slope close to 0.5.

damped. Also, the turbulence would be very anisotropic and there is no reason to expect a Kolmogorov like scaling in such a case. In addition to this, the moments of velocity differences as well as their pdf's show a behavior very different from three-dimensional results. A Kolmogorov scaling is expected, regardless of the spatial dimension (in two and three dimensions, isotropy of the flow is also necessary), as long as the energy flux is constant for a certain range of scales. This has been shown recently in a study of the 1d Burgers equation, driven with a particular noise, which finds that the system displays a scaling for the energy density spectrum similar to the Kolmogorov scaling for three-dimensional turbulence [1,2]. These studies have also found strong anomalies for velocity increments. In particular it was found that the high order moments of velocity increments show a strong deviation from Kolmogorov scaling. In particular, these moments varied linearly with the separation r for all moments starting from the third moment, while the second moment followed a dependence close to a Kolmogorov scaling (*i.e.* it varied as $r^{2/3}$). Our observations agree with these findings although the second moment has a weaker dependence in our case; we find roughly 0.5 instead of 2/3. Another intriguing similarity comes from the shape of the pdf of these velocity increments, namely an exponential part for positive increments and a power law part for the negative increments was seen in numerical simulations [1] as well as in exact calculations [3]. Our observations and measurements are in qualitative agreement with these preditions.

Considering the data reported here, it is reasonable to model our system (in the rough film regime) as a one-dimensional "compressible" fluid. In order to justify the one dimensionality of the system used in this experiment, it is necessary to assume that the column of water keeps its axial symetry making fluctuations in the angular direction negligeable (this component is very difficult to measure with our methods and was therefore not studied). Also, since the thickness of the film is small, less than 0.05 cm, velocity fluctuations in the radial direction are strongly damped because of friction against the wire. In this sense, the most important dynamics is in the longitudinal direction. Also, experimentally it was found that the transverse velocity fluctuations are much smaller than the longitudinal velocity fluctuations. The Navier Stokes equation governing the flow can then be approximated by its one-dimensional equivalent for the longitudinal component. In this approximation the fluid can be considered as a compressible fluid in one-dimension since the thickness of the column changes; the density of the fluid is proportional to the cross-section of the column: $\rho_h = \pi\rho(h^2 - r_w^2)$ where ρ_h is the one-dimensional density, ρ is the density of water in this case, h is the fluctuating radius of the column, and r_w the radius of the wire. The conservation of mass can be written as: $d\rho_h/dt + d/dz(v\rho_h) = 0$.

The one-dimensional Navier Stokes equation governing the longitudinal component of the velocity v_z, which for simplicity we denote as v, can be written in a similar way as was done for modeling the buildup and formation of necks by detaching drops of viscous fluids [6]:

$$dv/dt + vdv/dz = T/\rho(h_0^{-2}dh/dz + d^3h/dz^3) + \nu d^2v/dz^2 + F_w + g.$$

$F_w(= \nu d^2v/dr^2)$ is a term taking into account the friction due to the wire (with ν the kinematic viscosity of water and r is in the radial direction) and depends critically on the shape of the velocity profile in the flowing film. This term provides a general damping of the dynamics and acts to inhibit velocity fluctuations perpendicular to the wire. Despite this damping term large velocity fluctuations and a transition to turbulence is still seen. An estimate of the ratio of the inertial forces to this viscous damping term gives a number of about 1000 at the highest velocities used (for this estimate we use a mean velocity of 200 cm/s, a length of 0.05 cm for a typical column radius and the kinematic viscosity of water). This is the three-dimensional Reynolds number for the experiment. The number is small for fully developed 3d turbulence to exist but large enough to allow the inertial forces to have an important role in the dynamics. h_0 is the average radius of the column, T is the surface tension of the water-air interface and the term proportional to this tension takes into account the fluctuations of the thickness of the column. This term is an approximation for small undulations [7]. However since in this case the undulations can be large compared to the mean thickness of the column, higher order terms should be taken into account as was done in the study of detaching drops [6]. The second term on the right hand side is the usual friction term due to the fluid viscosity. g is the gravitational acceleration.

This system of equations reduces the problem to a one-dimensional one: *a compressible fluid in one dimension.*

The compressibility stems from the instability of the column to thickness variations and it is clear from these equations that the fluctuations in the thickness of the column are coupled to the fluctuations of the velocity. The interplay between the instabilty of the column to undulations and the velocity must be an important turbulence generating mechanism giving rise to the chaotic behavior observed for the velocity. This problem is the one-dimensional analogue of film flow down flat surfaces for which much experimental and theoretical work has been devoted since the pioneering work of P.L. Kapitza and S.P. Kapitza (see Ref. [8] for a review). Some similarities exist between the two systems: the instability is convective and a turbulent region is observed downstream for high flow rates. A major difference between the two systems comes from the intrinsic instability of the column to drop formation as opposed to the stability of a flat film flowing down a surface.

Conclusion

We have presented measurements of the velocity fluctuations of a thin column of water streaming down a wire. This column is smooth near the entrance, becomes unstable further down stream, where a rough film regime is observed, and breaks up into drops farther away. The results show that in the rough film regime the fluctuations of the longitudinal velocity are much more important than fluctuations in the transverse velocity. The power spectrum of the longitudinal velocity shows a scaling law as a function of the frequency at high velocities. Using the frozen turbulence assumption, we find that the energy density spectrum obeys a scaling law similar to the Kolmogorov $-5/3$ law for three-dimensional turbulence. It is argued that the system can be modeled as a one-dimensional compressible fluid where the compressibilty stems from the instability of the column to thickness fluctuations. While at this point, the origin of the scaling law we observe experimentally is not clear, it remains a possibility that in the rough film regime the thickness fluctuations of the column are acting as a source of noise in analogy with recent numerical and theoretical work on the stochastic Burgers equation [1–3]. A crucial issue remains unsolved, namely why the term taking the fluctuations of the column thickness into account should act as a source of spatially correlated noise.

The authors are grateful to D. Bonn for a critical reading of the manuscript, and to M. Adda-Bedia, J. Meunier, and V. Hakim for discussions.

References

1. A. Chekhlov, V. Yakhot, Phys. Rev. E **51**, R2739 (1995); A. Chekhlov, V. Yakhot, Phys. Rev. E **52**, 5681 (1995).
2. F. Hayot, C. Jayaprakash, Phys. Rev. E **54**, 4681 (1996); F. Hayot, C. Jayaprakash, Phys. Rev. E **56**, 227 (1997).
3. A. Polyakov, Phys. Rev. E **52**, 6183 (1995).
4. J.P. Bouchaud, M. Mézard, G. Parisi, Phys. Rev. E **52**, 3656 (1995).
5. H. Kellay, X.L. Wu, W.I. Goldburg, Phys. Rev. Lett. **74**, 3975 (1995).
6. X.D. Shi, M.P. Brenner, S.R. Nagel, Science **265**, 219 (1994); J. Eggers, T.F. Dupont, J. Fluid Mech. **262**, 205 (1994).
7. H. Lamb, *Hydrodynamics* (Cambridge University press, 6th edition, paperback edition 1993), p. 473.
8. H.C. Chang, Ann. Rev. Fluid Mech. (1994), p. 103.